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A CHARACTERIZATION OF THE INFINITESIMAL CONFORMAL TRANSFORMATIONS ON TANGENT BUNDLES

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Abstract. Here, we present a new complete lift metric for which every infinitesimal fiber-preserving conformal transformation on the tangent bundle induces an infinitesimal projective transformation on the base manifold. Moreover, this correspondence gives rise to a homomorphism between Lie algebras. Also, we introduce an almost product structure on the tangent bundle and show that it is a product structure if and only if the corresponding Riemannian metric is of constant curvature.

1. Introduction

Let M be a Riemannian manifold, and ϕ be a transformation of M. Then, ϕ is called a projective transformation if it preserves the geodesics, where each geodesic should be confounded with a subset of M by neglecting its affine parameter. Furthermore, ϕ is called an affine transformation, if it preserves the Riemannian connection. We may also speak of local projective and affine transformation. Then, we remark that a (local) affine transformation may be characterized as a (local) projective transformation which preserves the affine parameter of geodesics.

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Let V be a vector field on M, and consider the local one-parameter group $\{\phi_t\}$ of the local transformations of M generated by V. Then, V is called an infinitesimal projective (respectively affine) transformation, if each ϕ_t is a local projective (respectively affine) transformation. By a complete infinitesimal projective transformation, we mean an infinitesimal projective transformation which generates a (global) one-parameter group of projective transformations.

Let TM be the tangent bundle of M, and ϕ be a transformation of TM. Then, ϕ is called a fibre-preserving transformation, if it takes fibres to fibres. Let X be a vector field on TM, and consider the local one-parameter group $\{\phi_t\}$ of the local transformations of TM generated by X. Then, X is called an infinitesimal fibre-preserving transformation on TM, if each ϕ_t is a local fibre-preserving transformation of TM. Clearly, an infinitesimal fiber-preserving transformation on TM induces an infinitesimal transformation in the base space M. Let $\bar{g}$ be a (pseudo)-Riemannian metric of TM. An infinitesimal fiber-preserving transformation X on TM is said to be an infinitesimal fiber-preserving conformal transformation, if there exists a scalar $\bar{\rho}$ on TM such that $\pounds_X \bar{g} = 2\bar{\rho}\bar{g}$, where $\pounds_X$ denotes the Lie derivation with respect to X.

Let P be an endomorphism of the tangent bundle TM satisfying $P^2 = I$, where $I = identity$. Then, P defines an almost product structure on M. If g is a metric on M such that $g(PX, PY) = g(X, Y)$ for arbitrary vector fields X and Y on M, then the triple (M, g, P) defines a (pseudo)-Riemannian almost product structure.

Here, we define a new kind of (pseudo)-Riemannian metric G on TM and introduce the natural almost product structure P on M. The main purpose is to investigate some relations between the Lie algebra of infinitesimal fiber-preserving conformal transformations of the tangent bundle TM and the Lie algebra of infinitesimal projective transformations of M.

Throughout the paper, everything is C^∞, and Riemannian manifolds are connected with $dimM > 1$. Also, we suppose $\widetilde{TM} = TM - \{0\}$.

2. **Complete lift metric**

Let (M, g) be an n-dimensional (pseudo)-Riemannian manifold and ∇ its Levi-Civita connection. In a local chart $(U, (x^i))$, we set $g_{ij} = g(\partial_i, \partial_j)$, where $\partial_i := \frac{\partial}{\partial x^i}$ and we denote by ${\Gamma_{jk}}^i$ the corresponding

Christoffel symbols. Let $(x^i, y^i) \equiv (x, y)$ be the local coordinates on the manifold TM projected on M by τ. The indices $i, j, k, ...$ are taken from 1 to n.

The functions $N_j{}^i(x, y) := \Gamma_{jk}^{\ i}(x)y^k$ are the local coefficients of a nonlinear connection, that is, the local vector fields $\delta_i = \partial_i - N_i{}^k(x, y)\partial_{\bar{k}}$, where $\partial_{\bar{k}} = \frac{\partial}{\partial y^k}$ spans a distribution on TM called horizontal, which is supplementary to the vertical distribution $u \to V_u TM = ker(\tau_*)_u$, where $u \in TM$. Denote by $u \to H_u TM$ the horizontal distribution and let $\{\delta_i, \partial_{\bar{i}}\}$ be the basis adapted to the decomposition $T_u TM = H_u TM \oplus V_u TM$, where $u \in TM$. The dual basis of it is $\{dx^i, \delta y^i\}$ with $\delta y^i = dy^i + N_k{}^i(x, y)dx^k$.

We can easily prove the following lemma.

Lemma 2.1. The Lie brackets satisfy the followings:

$$[\delta_i, \delta_j] = y^r K_{jir}{}^m \partial_{\bar{m}},$$
$$[\delta_i, \partial_{\bar{j}}] = \Gamma_{ji}{}^m \partial_{\bar{m}},$$
$$[\partial_{\bar{i}}, \partial_{\bar{j}}] = 0,$$

where $K_{jir}{}^m$ denotes the components of the curvature tensor of M.

The complete metric on TM is:

$$G_C = 2g_{ij}(x)dx^i \delta y^j.$$

If we define $g_{ij}(x)$ as the components $h_{ij}(x, y)$ of a generalized Lagrange metric([3]), then we get a complete metric,

$$G(x, y) = 2h_{ij}(x, y)dx^i \delta y^j.$$

In particular, $h_{ij}(x, y)$ could be a deformation of $g_{ij}(x)$, a case studied by Anastasiei in [2].

Here, we consider the metric G with $h_{ij}(x, y)$ to be the special deformation of $g_{ij}(x)$ of the form:

$$h_{ij}(x, y) = a(L^2)g_{ij}(x),$$

where $L^2 = g_{ij}(x)y^i y^j, y_i = g_{ij}(x)y^j$ and $a : \mathrm{Im}(L^2) \subseteq \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ with $a > 0$.

3. **Almost product structures on TM**

Let P be an endomorphism of the tangent bundle TM given in the adapted basis $\{\delta_i, \partial_{\bar{i}}\}$ by

$$P(\delta_i) = \alpha\partial_{\bar{i}}, \quad P(\partial_{\bar{i}}) = \beta\delta_i,$$

where α and β are functions on TM to be determined. Then, we have,

$$P^2(\delta_i) = \alpha\beta\delta_i, \quad P^2(\partial_{\bar{i}}) = \beta\alpha\partial_{\bar{i}},$$

i.e., the condition $P^2 = I$ leads to $\alpha\beta = 1$.

With the above condition, we conclude that $G(P(X), P(Y)) = G(X, Y)$. Then, the pair (G, P) is an almost product structure on TM.
Put

$$\alpha = \frac{1}{a}, \qquad \beta = a.$$

Then, we have,

$$P(\delta_i) = \frac{1}{a}\partial_{\bar{i}}, \quad P(\partial_{\bar{i}}) = a\delta_i. \tag{3.1}$$

Substitution $a \longrightarrow \frac{a}{L}$, then (1) is unified to:

$$P_{a,L}(\delta_i) = \frac{L}{a}\partial_{\bar{i}}, \quad P_{a,L}(\partial_{\bar{i}}) = \frac{a}{L}\delta_i. \tag{3.2}$$

The metric G takes the form,

$$G_{a,L}(x, y) = 2\frac{a}{L}g_{ij}(x)dx^i\delta y^j. \tag{3.3}$$

If $a = \frac{L}{\sqrt{1+L^2}}$, then the relations (3.2) and (3.3) turn to:

$$P_L(\delta_i) = \sqrt{1 + L^2}\partial_{\bar{i}}, \quad P_L(\partial_{\bar{i}}) = \frac{1}{\sqrt{1 + L^2}}\delta_i, \tag{3.4}$$

$$G_L(x, y) = \frac{2}{\sqrt{1 + L^2}}g_{ij}(x)dx^i\delta y^j. \tag{3.5}$$

If $a = c$, where c is a constant scalar, then (3.2) and (3.3) take the form,

$$P_{c,L}(\delta_i) = \frac{L}{c}\partial_{\bar{i}}, \quad P_{c,L}(\partial_{\bar{i}}) = \frac{c}{L}\delta_i, \tag{3.6}$$

$$G_{c,L}(x, y) = 2\frac{c}{L}g_{ij}(x)dx^i\delta y^j. \tag{3.7}$$

Here, we consider the almost product structures (G, P), $(G_{a,L}, P_{a,L})$, (G_L, P_L) and $(G_{c,L}, P_{c,L})$.

In order to find conditions for the above almost product structures to be product structures, we have to put zero for the Nijenhuis tensor field of $P = P_a, P_{a,L}, P_L, P_{c,L}$,

$$N_P(X,Y) = [PX,PY] - P[PX,Y] - P[X,PY] + [X,Y], \quad X,Y \in \chi(M).$$

By a simple calculation, we have the following results.

Proposition 3.1. *In the adapted basis we have the unique decomposition,*

$$N_P(\delta_i,\delta_j) = (N_P)_{ij}^s\delta_s + (N_P)_{ij}^{\bar{s}}\partial_{\bar{s}}$$
$$N_P(\delta_i,\partial_{\bar{j}}) = (N_P)_{i\bar{j}}^s\delta_s + (N_P)_{i\bar{j}}^{\bar{s}}\partial_{\bar{s}}$$
$$N_P(\partial_{\bar{i}},\partial_{\bar{j}}) = (N_P)_{\bar{i}\bar{j}}^s\delta_s + (N_P)_{\bar{i}\bar{j}}^{\bar{s}}\partial_{\bar{s}}$$

where,

$$(N_P)_{ij}{}^{\bar{s}} = y^a K_{jia}{}^s + \frac{2a'}{a^3}(y_j\delta_i^s - y_i\delta_j^s), \quad (N_P)_{ij}{}^s = 0$$
$$(N_P)_{i\bar{j}}{}^s = -a^2\{y^a K_{jia}{}^s + \frac{2a'}{a^3}(y_j\delta_i^s - y_i\delta_j^s)\}, \quad (N_P)_{i\bar{j}}{}^{\bar{s}} = 0$$
$$(N_P)_{\bar{i}\bar{j}}{}^{\bar{s}} = a^2\{y^a K_{jia}{}^s + \frac{2a'}{a^3}(y_j\delta_i^s - y_i\delta_j^s)\}, \quad (N_P)_{\bar{i}\bar{j}}{}^s = 0.$$

Lemma 3.2. *P is a product structure on $\widetilde{TM}$ if and only if we have,*

$$y^a K_{jia}{}^s = -\frac{2a'}{a^3}(y_i\delta_j^s - y_j\delta_i^s). \tag{3.8}$$

From (3.8) and $y_i = g_{ia}y^a$, we obtain following equation,

$$K_{jia}{}^s = -\frac{2a'}{a^3}(g_{ia}\delta_j^s - g_{ja}\delta_i^s). \tag{3.9}$$

From (3.9), we conclude following theorem.

Theorem 3.3. *Let a be a function such that $a'(t) = \frac{1}{2}ka^3(t)$, where k is a constant. Then the almost product structure P is a product structure on $\widetilde{T}M$ if and only if the Riemannian space (M,g) is of constant curvature $-k$.*

For example, if we suppose $a(t) = \frac{1}{\sqrt{t}}$, then we have $a'(t) = -\frac{1}{2}a^3(t)$. In this case, the value of k in Theorem 3.3 is equal to 1. Taking $N_{P_{a,L}} =$

0, we conclude:

$$K_{jia}{}^{s} = -\frac{2a'L^2 - a}{a^3}(g_{ia}\delta_j^s - g_{ja}\delta_i^s). \tag{3.10}$$

From (3.10), we have the following result.

Theorem 3.4. *Let a be a function such that $\frac{2a'L^2-a}{a^3} = k$, where k is a constant. Then, the almost product structure $P_{a,L}$ is a product structure on $\widetilde{TM}$ if and only if the Riemannian space (M, g) is of constant curvature $-k$.*

Taking $N_{P_L} = 0$, we conclude:

$$K_{jia}{}^{s} = -(g_{ia}\delta_j^s - g_{ja}\delta_i^s). \tag{3.11}$$

From (3.11), we have the following theorem.

Theorem 3.5. *The almost product structure P_L is a product structure on $\widetilde{TM}$ if and only if the Riemannian space (M, g) is of constant curvature -1.*

Taking $N_{P_{c,L}} = 0$, we conclude:

$$K_{jia}{}^{s} = -\frac{1}{c^2}(g_{ia}\delta_j^s - g_{ja}\delta_i^s). \tag{3.12}$$

From (3.12), we get the following theorem.

Theorem 3.6. *The almost product structure $P_{c,L}$ is a product structure on $\widetilde{TM}$ if and only if the Riemannian space (M, g) is of constant curvature $-\frac{1}{c^2}$.*

4. **Infinitesimal Conformal Transformation**

Here, we consider the infinitesimal conformal transformations of the tangent bundles over Riemannian manifolds. First of all, we recall that the vector field X on TM with components $(v^h, v^{\bar{h}})$ is a fiber-preserving vector field if and only if v^h are functions on M (see [5]).

Proposition 4.1. *Let X be a fiber-preserving vector field on TM. Then, the Lie derivative $\pounds_X\delta_h$, $\pounds_X\partial_{\bar{h}}$, $\pounds_X dx^h$ and $\pounds_X\delta y^h$ are given as follow:*

$$\begin{array}{lll} (1)\,\pounds_X\delta_h & = & -\partial_h v^a\delta_a + \{y^b v^c K_{hcb}{}^a - v^{\bar{b}}\Gamma_{bh}{}^a - \delta_h(v^{\bar{a}})\}\partial_{\bar{a}}, \\ (2)\,\pounds_X\partial_{\bar{h}} & = & \{v^b\Gamma_{hb}{}^a - \partial_{\bar{h}}(v^{\bar{a}})\}\partial_{\bar{a}}, \\ (3)\,\pounds_X dx^h & = & \partial_m v^h dx^m, \\ (4)\,\pounds_X\delta y^h & = & -\{y^b\ v^c K_{mcb}{}^h - v^{\bar{b}}\Gamma_{bm}{}^h - \delta_m(v^{\bar{h}})\}dx^m \\ & & -\{v^b\Gamma_{mb}{}^h - \partial_{\bar{m}}(v^{\bar{h}})\}\delta y^m. \end{array}$$

Proof. Proof of this Theorem is similar to proof of the Proposition 2.2 of Yamauchi [5]. □

Proposition 4.2. *The Lie derivatives $\pounds_X G$ is in the following form:*

$$\pounds_X G = -2a(L^2)g_{im}\{y^b v^c K_{jcb}{}^m - v^{\bar{b}}\Gamma_{bj}{}^m - \delta_j(v^{\bar{m}})\}dx^i dx^j$$
$$+2a(L^2)\{2\bar{\varphi}g_{ij}+\pounds_V g_{ij}-g_{im}\nabla_j v^m+g_{im}\partial_{\bar{j}}(v^{\bar{m}})\}dx^i\delta y^j,$$

where $\bar{\varphi} = v^{\bar{h}}y_h\frac{a'(L^2)}{a(L^2)}$.

Proof. From the definition of Lie derivative we have:

$$\pounds_X G = \pounds_X(a(L^2))(2g_{ij}dx^i\delta y^j) + a(L^2)\pounds_X(2g_{ij}dx^i\delta y^j). \tag{4.1}$$

By Proposition 4.1, we conclude the following result:

$$\pounds_X(2g_{ij}dx^i\delta y^j) = -2g_{im}\{y^b v^c K_{jcb}{}^m - v^{\bar{b}}\Gamma_{bj}{}^m - \delta_j(v^{\bar{m}})\}dx^i dx^j.$$
$$+2\{\pounds_V g_{ij} - g_{im}\nabla_j v^m + g_{im}\partial_{\bar{j}}(v^{\bar{m}})\}dx^i\delta y^j. \tag{4.2}$$

Also, it is obvious that:

$$\pounds_X(a(L^2)) = X(a(L^2)) = 2v^{\bar{h}}y_h a'(L^2). \tag{4.3}$$

Putting (4.3) and (4.2) in (4.1), we have the proof. □

Let X be an infinitesimal fibre-preserving conformal transformation on TM with metric G. Then, there exists a scalar function $\bar{\rho}$ on TM such that

$$\pounds_X G = 2\bar{\rho}G.$$

From proposition 4.2, we have,

$$2\bar{\varphi}g_{ij} + \pounds_V g_{ij} - g_{im}\nabla_j v^m + g_{im}\partial_{\bar{j}}(v^{\bar{m}}) = 2\bar{\rho}g_{ij}, \tag{4.4}$$

and

$$g_{im}\{y^b v^c K_{jcb}{}^m - v^{\bar{b}}\Gamma_{bj}{}^m - \delta_j(v^{\bar{m}})\} + g_{jm}\{y^b v^c K_{icb}{}^m - v^{\bar{b}}\Gamma_{bi}{}^m - \delta_i(v^{\bar{m}})\} = 0. \tag{4.5}$$

The (4.4) can be written as:

$$\pounds_V g_{ij} - g_{im}\nabla_j v^m + g_{im}\partial_{\bar{j}}(v^{\bar{m}}) = 2(\bar{\rho} - \bar{\varphi})g_{ij}.$$

Put $\bar{\Omega} = \bar{\rho} - \bar{\varphi}$. Then, we conclude following relation:

$$\pounds_V g_{ij} - g_{im}\nabla_j v^m + g_{im}\partial_{\bar{j}}(v^{\bar{m}}) = 2\bar{\Omega} g_{ij}. \tag{4.6}$$

Proposition 4.3. *The scalar function $\bar{\Omega}$ on TM depends only on the variables (x^h) with respect to the induced coordinates (x^h, y^h).*

Proof. Applying $\partial_{\bar{k}}$ to the both sides of the equation (4.6), then we have,

$$g_{im}\partial_{\bar{k}}\partial_{\bar{j}}(v^{\bar{m}}) = 2\partial_{\bar{k}}(\bar{\Omega})g_{ij}.$$

By interchanging j and k in the above equation, we get,

$$\partial_{\bar{k}}(\bar{\Omega})g_{ij} = \partial_{\bar{j}}(\bar{\Omega})g_{ik}.$$

It follows that

$$(n-1)\partial_{\bar{k}}(\bar{\Omega}) = 0.$$

This means that the scalar function $\bar{\Omega}$ on TM depends only on the variables (x^h) with respect to the induced coordinates (x^h, y^h). □

Thus, we can regard $\bar{\Omega}$ as a function on M. In the following, we write Ω instead of $\bar{\Omega}$.

Also, let X be an infinitesimal fibre-preserving conformal transformation on TM with metric $G_{a,L}$ and scalar function $\bar{\rho}_{a,L}$. Then, we have $\Omega_{a,L} = \bar{\rho}_{a,L} - \bar{\varphi}_{a,L}$, where,

$$\bar{\varphi}_{a,L} = v^{\bar{h}} y_h \left(\frac{L^2 a' - \frac{1}{2}a}{L^3}\right).$$

Similarly, for G_L we have $\Omega_L = \bar{\rho}_L - \bar{\varphi}_L$ with

$$\bar{\varphi}_L = -\frac{v^{\bar{h}} y_h}{2(1+L^2)\sqrt{1+L^2}},$$

and for $G_{c,L}$ we have $\Omega_{c,L} = \bar{\rho}_{c,L} - \bar{\varphi}_{c,L}$ with

$$\bar{\varphi}_{c,L} = -\frac{v^{\bar{h}} y_h}{2L^3}.$$

From (4.6) and proposition 4.3, $\partial_{\bar{j}}(v^{\bar{m}})$ depends only on the variables (x^h), and thus we can put

$$v^{\bar{m}} = y^a A^m{}_a + B^m, \tag{4.7}$$

where $A^m{}_a$ and B^m are certain functions depending only on the variable (x^h). Furthermore, we can easily show that $A^m{}_a$ and B^m are the components of a $(1,1)$ tensor field and a contravariant vector field on M, respectively.

Substituting (4.7) into (4.5), we have,

$$\nabla_j B_i + \nabla_i B_j = 0, \tag{4.8}$$

and

$$v^a(K_{jahi} + K_{iahj}) - \nabla_j A_{ih} - \nabla_i A_{jh} = 0, \tag{4.9}$$

where $B_i = g_{im}B^m$ and $A_{ih} = g_{im}A^m{}_h$.

Proposition 4.4. *If we put*

$$B = B^b \partial_b,$$

then the vector field B on M is an infinitesimal isometry of M.

Proof. From equation (4.8) we have,

$$\pounds_B g_{ij} = \nabla_j B_i + \nabla_i B_j = 0.$$

This shows B is an infinitesimal isometry on M. □

Proposition 4.5. *If we put*

$$V = v^h \partial_h,$$

then the vector field V on M is an infinitesimal projective transformation of M.

Proof. Substituating (4.7) into (4.6), it follows:

$$A_{ij} = 2\Omega g_{ij} - \nabla_i v_j. \tag{4.10}$$

Substituating (4.10) into (4.9), we obtain,

$$\pounds_V \Gamma_{ij}{}^h = \delta_i^h \Omega_j + \delta_j^h \Omega_i,$$

where $\Omega_i = \nabla_i \Omega$. This shows that V is an infinitesimal projective transformation on M.

Now, we consider the converse problem, that is, let M admits an infinitesimal projective transformation $V = v^h\partial_h$. Then, we have the following proposition.

Proposition 4.6. *The vector field X on TM defined by*

$$X = v^h\delta_h + y^r A^h{}_r\partial_{\bar{h}},$$

is an infinitesimal fibre-preserving conformal transformation on TM with metric G, where,

$$A^h{}_i = g^{hr}A_{ri}, \quad A_{ij} = \nabla_j v_i + 2\Omega g_{ij} - \pounds_V g_{ij}, \quad \Omega = \frac{1}{n+1}\nabla_r v^r,$$

$$\bar{\varphi} = \frac{a'(L^2)}{a(L^2)}y^r A^h{}_r y_h,$$

and $\bar{\rho} = \Omega + \bar{\varphi}$.

Proof. By proposition 4.2, it follows:

$$\begin{aligned}
\pounds_X G &= \pounds_X(2a(L^2)g_{ij}dx^i\delta y^j) \\
&= 2X(a(L^2)g_{ij})dx^i\delta y^j + 2a(L^2)g_{ij}(\pounds_X dx^i)\delta y^j \\
&\quad + 2a(L^2)g_{ij}dx^i(\pounds_X\delta y^j) \\
&= 4y^r A^h{}_r a'(L^2)y_h g_{ij}dx^i\delta y^j + 4a(L^2)\Omega g_{ij}dx^i\delta y^j \\
&\quad + 2a(L^2)y^r(v^b K_{bjri} + \nabla_j A_{ir})dx^i dx^j \\
&= 4a(L^2)(y^r A^h{}_r\frac{a'(L^2)}{a(L^2)}y_h + \Omega)g_{ij}dx^i\delta y^j \\
&\quad + 2a(L^2)y^r(v^b K_{bjri} + \nabla_j A_{ir})dx^i dx^j.
\end{aligned}$$

On the other hand, from (4.10), we have,

$$\begin{aligned}
\nabla_j A_{ir} &= g_{im}\nabla_j\nabla_r v^m + 2\Omega_j g_{ir} - (\pounds_V\Gamma_{ji}{}^m)g_{mr} - (\pounds_V\Gamma_{jr}{}^m)g_{im} \\
&= -v^b K_{bjri} + \Omega_j g_{ir} - \Omega_i g_{jr},
\end{aligned}$$

from which we obtain,

$$\pounds_X G = 2\bar{\rho}G.$$

Hence, X is an infinitesimal fibre-preserving conformal transformation on TM. □

Proposition 4.6 holds for TM with metric $G_{a,L}$ if we have,

$$\bar{\varphi}_{a,L} = v^{\bar{h}}y_h(\frac{L^2a' - \frac{1}{2}a}{L^3}).$$

Similarly, for G_L we have,

$$\bar{\varphi}_L = -\frac{v^{\bar{h}} y_h}{2(1+L^2)\sqrt{1+L^2}},$$

and for $G_{c,L}$ we have,

$$\bar{\varphi}_{c,L} = -\frac{v^{\bar{h}} y_h}{2L^3}.$$

Now, using propositions 4.3 to 4.6, we conclude the following theorem.

Theorem 4.7. *Let M be an n-dimensional Riemannian manifold, and TM be its tangent bundle with the metric G. Then, every infinitesimal fibre-preserving conformal transformation X on TM naturally induces an infinitesimal projective transformation V on M. Furthermore, the correspondence $X \longrightarrow V$ gives a homomorphism of the Lie algebra of infinitesimal fiber-preserving conformal transformations of TM onto the Lie algebra of infinitesimal projective transformations of M, and the kernel of this homomorphism is naturally homomorphic onto the Lie algebra of infinitesimal isometries of M.*

The above theorem holds for Pseudo-Riemannian metric $G_{a,L}, G_L$ and $G_{c,L}$.

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